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A CHARGE-SEPARATION MECHANISM FOR THE PRODUCTION OF POLAR AURORAS AND ELECTROJETS

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SANTA MONICA • CALIFORNIA

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PREFACE

The causes of polar auroras and electrojet current systems have been the subject of much conjecture by workers in the field of geophysics. In the present report, the author attempts to relate these phenomena to the effects of solar streams on the regions of geomagnetically trapped radiation.

This Memorandum, which extends the ideas presented in RM-2753-NASA, Geomagnetic Field Distortion by a Solar Stream as a Mechanism for the Production of Polar Aurora and Electrojets, represents part of a continuing study of the properties of charged particles and fields in space, and was prepared for publication in the Journal of Geophysical Research.

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ABSTRACT

A mechanism for charge separation in the geomagnetically trapped radiation is described which may account for some observed phenomena associated with the polar aurora and the electrojet current systems. Surfaces of constant number density may be separated from surfaces of constant integral invariant within the trapped radiation as a result of distortion of the geomagnetic field by solar streams. Drift separation of protons and electrons will follow, and for irregular distributions of plasma number density, electric fields will arise. A direct consequence of such polarization of the geomagnetically trapped radiation will be the polar-electrojet current systems.

The polar aurora arise where energetic particles are discharged from regions of excess charge within the geomagnetically trapped radiation. A model for the discharge of such auroral particles is discussed.

An interesting feature of the proposed mechanism is that the extreme thinness of auroral sheets appears to follow as a natural consequence of charge separation. This is shown analytically for a simple two-dimensional model of a trapped plasma.

CONTENTS

I. INTRODUCTION AND SUMMARY

In a letter to the editor of the J. Geophys. Research (Kern, 1961), the author suggested the possibility that solar-stream distortion of the geomagnetic field might lead to a situation in which charge separation occurs in trapped radiation, thus giving rise to polar-electrojet current systems. The physical basis for the mechanism suggested is adiabatic particle motion caused by eastward- or westward-directed geomagnetic field gradients. Such geomagnetic field gradients could be expected as a result of the interaction of the geomagnetic field, containing trapped plasma, and an enveloping ionized solar stream of comparatively low-energy particles.

Charge separation had been proposed earlier by Vestine (1960), Chamberlain, Kern, and Vestine (1960), and Kern and Vestine (1961) to account for some of the observed features of polar auroras and electrojet current systems. The present paper discusses the mechanism for charge separation outlined in the author's letter (Kern, 1961) and discussed by Chamberlain (1961) in connection with his auroral theory. In particular, the effects of increasing electric fields are studied. It is found that thin auroral sheets can be accounted for by the proposed charge separation. Discharge of auroral particles from regions of excess charge in the trapped plasma is also discussed.

Distortion of the geomagnetic field by ionized solar streams has been considered by many authors in relation to the occurrence of polar-electrojet current systems and aurora. The magnetic-storm theories of Chapman and Ferraro (1931, 1932, 1933) and Martyn (1951) require such distortion. Modification of the geomagnetic field by diamagnetic trapped-particle moments and by current systems has also been considered by Dessler and Parker (1959), Akasofu (1960) and others. Recent progress in the understanding of relationships between plasmas and magnetic fields has led to several pictures of the form of a solar-stream-distorted geomagnetic field (Johnson, 1960; Piddington, 1959, 1960; Parker, 1960).

The mechanism described in the author's letter (Kern, 1961) is similar in certain respects to that described by Martyn in his theory of magnetic storms and aurora (Martyn, 1951), based on Chapman and Ferraro's ring-current theory of magnetic storms (Chapman and Ferraro, 1931, 1932, 1933). In the present paper, however, the polarization of geomagnetically trapped radiation is ascribed to the displacement of particle density distributions with respect to surfaces of constant integral invariant.

Two alternative descriptions of the mechanism for charge separation in a geomagnetically trapped plasma are proposed here. First, plasma density irregularities somehow become inclined with respect to surfaces of constant integral invariant I; hence, the adiabatic drift of protons westward and electrons eastward along surfaces of constant I leads to charge separation. Or, second, a component of the geomagnetic field gradient arising from distortion of the geomagnetic field becomes oriented parallel to surfaces of constant plasma density.

Chamberlain (1961) has studied the effects of charge separation in geomagnetically trapped radiation in relation to auroral displays. His treatment is based on conservation of the total energy of a particle and relates change of electrostatic potential to equal and opposite changes in the transverse kinetic energy of trapped particles. In his model, acceleration of particles parallel to the magnetic field $\underset{\sim}{B}$ is associated with recovery of the original kinetic energy in motion along $\underset{\sim}{B}$. The treatment is applied to ray-type auroral structures. The present development will apply to auroral features of the sheet type; e.g., homogeneous arcs, draperies, and bands.

In Section II, charge separation resulting from adiabatic particle motions in the geomagnetically trapped radiation is examined. A two-dimensional model is analyzed, and it is found that the extremely thin sheets characteristic of auroral structures may be a natural consequence of such charge separation.

In Section III, the effects of an electric-field component parallel to the magnetic field $\underset{\sim}{B}$ are examined. The first particles discharged from a flux tube containing excess charge will be those with small magnetic moments. As discharge continues, the magnetic-moment distribution of particles within a flux tube will tend to have a lower bound. This lower bound of the magnetic-moment distribution in a region of excess charge is used as a basis for defining a model electric-field component parallel to $\underset{\sim}{B}$. A model electric field is defined which has just that magnitude and sense which permits those particles of one sign with lowest magnetic moment to move freely along a magnetic field line.

The motions of charged atmospheric particles up a flux tube in the presence of such an electric field are examined. Time scales for proton neutralization of an electron-charge excess are found to be long compared to electron discharge times. Thus electron precipitation can be anticipated. The lifetime of a proton-charge excess has been given by Chamberlain (1961) as about 1 sec.

Effects of the model electric field on mirror points of particles and on mirror-point energies are computed. Rates of change of the model electric field are computed for a constant flux of auroral particles into the atmosphere from a trapped plasma. The time scale for such an auroral flux is derived and is about 10^4 sec for a plasma density of 10 particles/cm^3 and a flux density in the atmosphere of about 10^9 particles/cm^2 sec. The same time scale applies to the growth of the electric field along $\mathbf{B}$. One significant result of the analysis is that particles mirroring at a given altitude will increase in energy as the display proceeds.

The relationships between charge separation in a geomagnetically trapped plasma and ionospheric current systems are discussed in Section IV. An estimate is made of the distortion of the geomagnetic field necessary to produce a typical westward-directed electrojet. It is concluded that moderate distortions would suffice in the presence of plasma-energy densities that are clearly less than the geomagnetic-field energy density in regions connected with the auroral zone.

Charge separation in the geomagnetically trapped radiation has implications for auroral morphology, and these are discussed in Section V. Correlations with observations are discussed, and a possible means of generating horseshoe auroral arcs is outlined. Section VI deals with a possible explanation of the tendencies of polar auroras and electrojet current systems to recur.

II. CHARGE SEPARATION AND POLARIZATION IN GEOMAGNETICALLY TRAPPED RADIATION

In the geomagnetically trapped radiation, there are two distinct types of adiabatic drift characterized by whether protons and electrons drift in the same or in opposite directions. $\underset{\sim}{E}\times\underset{\sim}{B}$ drift or any drift caused by forces that depend on the sign of electric charge is in the same direction for both protons and electrons. Inertial forces or those dependent on magnetic moment will lead to adiabatic drift of protons and electrons in opposite directions.

The familiar westward drift of trapped protons and eastward drift of trapped electrons due to the curvature and gradient of the geomagnetic field is given by

$$\underset{\sim}{v}_B = - \frac{\nabla_{\perp} B \times \underset{\sim}{B}}{e\, B^3}\, W(1 + \cos^2 \alpha) \tag{1}$$

where $\nabla_{\perp} B$ is the component of ∇B transverse to $\underset{\sim}{B}$, e is the charge of the particle in emu, W is the kinetic energy of the particle in ergs, and α is the pitch angle of particle motion with respect to $\underset{\sim}{B}$ (Spitzer, 1956). In a plasma with an irregular number density, this kind of drift can lead to charge separation with associated electric fields. Figure 1 shows the result of this kind of charge separation in a bounded sheet of plasma.

In applying the idea of charge separation to the geomagnetically trapped radiation, distortion of the field-plasma geometry must be introduced in such a way that the drift velocity $\underset{\sim}{v}_B$ is not simply parallel to surfaces of constant number density. Figure 2 shows a configuration in which a component of $\nabla_{\perp} B$ (denoted $\nabla_{\varphi} B$) is parallel to surfaces of constant number density. This component of $\nabla_{\perp} B$ leads to charge separation and an electric field directed from the separated proton sheet toward the electron sheet.

It can be seen that if the transverse electric field arising from charge separation is increasing, the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift of both protons and electrons will be accelerated. Chandrasekhar (1960) shows that the adiabatic character of the particle motion is preserved for this situation if such a polarization drift velocity $\underset{\sim}{v}_p$ is introduced that

$$\underset{\sim}{v}_p = (m/eB^2) \frac{\partial \underset{\sim}{E}}{\partial t} \tag{2}$$

where m is the particle mass, e is the electronic charge in emu, B is the magnetic field, and $\frac{\partial \underset{\sim}{E}}{\partial t}$ is the rate of change of the electric field in emu/sec. This drift exactly compensates for the increase in kinetic energy arising from acceleration of the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift by a simultaneous decrease in electrostatic potential energy. Note that this drift is independent of particle energy and is much faster for protons than electrons, because of the larger proton mass. For a trapped plasma of the kind discussed here, the polarization drift v_p opposes the magnetic drift v_B. A two-dimensional plasma model will therefore be described for which the electric-field variation and its effects can be calculated.

Consider a two-dimensional plasma, such as that indicated in Fig. 1, for which surfaces of constant plasma density contain magnetic field lines. Suppose a magnetic field gradient $\nabla_{\perp} B$ exists transverse to the magnetic field $\underset{\sim}{B}$. The drift motions of protons and electrons in the plasma that are due to this gradient will be supposed to have a component perpendicular to surfaces of constant plasma density. In order to simplify the calculations, it will be assumed that the proton drift is much greater than that of the electrons. This would follow from (1) if the electrons' kinetic energies were very much less than those of the protons. If this is the case, motion of a proton can be specified locally by the drift velocity $\underset{\sim}{v}_d = \underset{\sim}{v}_B - \underset{\sim}{v}_p$, where $\underset{\sim}{v}_B$ and $\underset{\sim}{v}_p$ are given by Eqs. (1) and (2), respectively. For the two-dimensional geometry considered here, Gauss' theorem can be applied to obtain the electric field at any point inside the plasma. Here the velocity $\underset{\sim}{v}_B$ will be considered to be the component of the gradient drift velocity that is perpendicular to surfaces of constant plasma density. Such drift would result from a component of $\nabla_\perp B$ parallel to the surfaces of constant plasma density.

Right-handed coordinate axes x, y, and z are taken so that the z axis is in the direction of the magnetic field $\underset{\sim}{B}$. Surfaces of constant plasma density are taken parallel to the x-z plane, and the y-axis is in the direction of the plasma density gradient $n(y)$. The plasma number density is taken as varying from zero to some value $n(y)$ as y goes from $-\infty$ to y. A gradient in $\underset{\sim}{B}$ in the -x direction will lead to proton drift v_B in the -y direction. The polarization drift v_p of a proton will be in the +y direction; that is, v_p is in the direction of increasing $n(y)$. The total drift velocity $v_d = v_B - v_p$ will be in the -y direction if $v_B > v_p$, and in the +y direction if $v_p > v_B$. The drift v_B is in such a direction that protons will tend to separate out of the neutral plasma, as shown in Fig. 1. On the opposite side of the neutral-plasma feature, electrons will separate out. An electric field E will appear in the +y direction.

Now the rate of change of space charge per unit volume resulting from the drift velocity v_d is given by

$$\frac{\partial \rho}{\partial t} = e\int_0^{\infty} v_d(W)\, \frac{\partial F(\epsilon,y)}{\partial y}\, d\epsilon \tag{3}$$

where $v_d(W)$ is the total drift velocity, which is a function of proton kinetic energy W, and $F(\epsilon,y)$ is a distribution function for total proton energy ϵ, so that

$$\int_0^{\infty} F(\epsilon,y)\, d\epsilon = n(y).$$

The total energy ϵ is invariant for a given proton. The kinetic energy W will vary with the development of electrostatic potential V, so that $W = \epsilon - eV$.

Now for the geometry selected $\nabla \cdot \underset{\sim}{E} = 4\pi c^2 \rho$ (in emu) reduces to

$$\frac{\partial E}{\partial y} = 4\pi c^2 \rho$$

and hence

$$\frac{\partial^2 E}{\partial t \partial y} = 4\pi c^2 \frac{\partial \rho}{\partial t}$$

From Eq. (3) it follows that

$$\frac{\partial^2 E}{\partial y \partial t} = 4\pi e c^2 \int_0^{\infty} \left[v_B(W) - v_p \right] \frac{\partial F(\epsilon,y)}{\partial y}\, d\epsilon \tag{4}$$

Substituting from Eqs. (1) and (2) and noting that $W = \epsilon - eV$, Eq. (4) can be written

$$\frac{\partial^2 E}{\partial y \partial t} = 4\pi e c^2 \beta \int_0^\infty (\epsilon - eV) \frac{\partial F(\epsilon,y)}{\partial y} \, d\epsilon - \frac{4\pi m c^2}{B^2} \int_0^\infty \frac{\partial E}{\partial t} \frac{\partial F(\epsilon,y)}{\partial y} \, d\epsilon$$

where, from Eq. (1), $\beta = v_B/W$ = constant. Note that $\frac{\partial E}{\partial t}$ and V are not explicitly functions of ϵ. Also

$$\int_0^\infty \frac{\partial F(\epsilon,y)}{\partial y} \, d\epsilon = \frac{\partial n}{\partial y}$$

while, if ϵ does not depend on y,

$$\int_0^\infty \frac{\partial F(\epsilon,y)}{\partial y} \, d\epsilon = \bar{\epsilon} \frac{\partial n}{\partial y}$$

where $\bar{\epsilon}$ is the average total particle energy. Hence, Eq. (4) can be written

$$\frac{\partial^2 E}{\partial y \partial t} = 4\pi e c^2 \beta (\bar{\epsilon} - eV) \frac{\partial n}{\partial y} - \frac{4\pi m c^2}{B^2} \frac{\partial n}{\partial y} \frac{\partial E}{\partial t}$$

or

$$\frac{\partial^2 E}{\partial y \partial t} + \frac{4\pi m c^2}{B^2} \frac{\partial n}{\partial y} \frac{\partial E}{\partial t} = 4\pi e c^2 \beta (\bar{\epsilon} - eV) \frac{\partial n}{\partial y} \tag{6}$$

In order to solve this differential equation, the condition that $eV \ll \bar{\epsilon}$ will be imposed. This will be valid initially. For later times, the growth of the electric field will be retarded as eV becomes significant with respect to the mean particle energy. If $eV \ll \bar{\epsilon}$ the right side of (6) can be written $4\pi e c^2 \bar{v}_B \frac{\partial n}{\partial y}$, where $\bar{v}_B$ is the mean drift velocity due to the magnetic field gradient. Since the energy distribution is taken as independent of y, $\bar{v}_B$ is independent of y.

Equation (6), then, has the solution

$$\frac{\partial}{\partial y}\left[\frac{\partial E}{\partial t}\, e^{\gamma n}\right] = 4\pi e c^2\, \bar{v}_B \frac{\partial n}{\partial y}\, e^{\gamma n}$$

where $\gamma = 4\pi mc^2/B^2$. This equation can now be integrated with respect to y from some point outside the region of the plasma density feature (where both n and $\frac{\partial E}{\partial t}$ are zero) to y. The result is

$$\frac{\partial E}{\partial t}\, e^{\gamma n} = (4\pi e c^2\, \bar{v}_B/\gamma)(e^{\gamma n} - 1)$$

Hence the rate of change of the electric field is given by

$$\frac{\partial E}{\partial t} = \frac{4\pi e c^2 \bar{v}_B}{\gamma}\,(1 - e^{-\gamma n}) \tag{7}$$

where e is the proton charge in emu, c is the velocity of light, $\bar{v}_B$ is the mean drift velocity for protons in the negative y-direction resulting from a transverse magnetic-field gradient, n is the neutral plasma number density, and $\gamma = 4\pi mc^2/B^2$, where m is the proton mass and B is the value of the magnetic field. This expression is seen as valid initially. As mentioned above, the development of an electrostatic potential field, V, will modify the drift motion, effectively reducing the mean drift velocity $\bar{v}_B$. This will lead to a decrease in $\frac{\partial E}{\partial t}$. The electric field does not, therefore, increase without limit.

The rate of change of charge density ρ associated with this distribution of electric field can be computed from $\frac{\partial E}{\partial y} = 4\pi c^2 \rho$, which gives

$$\frac{\partial \rho}{\partial t} = (1/4\pi c^2)\,\frac{\partial^2 E}{\partial y\, \partial t}$$

or, from (7)

$$\frac{\partial\rho}{\partial t} = e\bar{v}_B \, e^{-\gamma n} \, \frac{dn}{dy} \qquad (8)$$

This expression gives the time variation of space charge ρ subject to the same restrictions as discussed for $\frac{\partial E}{\partial t}$ as given by (7). The space charge ρ is initially distributed very near the edges of any assumed plasma density feature. The extreme thinness of such a space charge distribution can be emphasized by considering an example.

For a plasma of number density n, the value of γ for protons is about $1.87 \times 10^{-2}\ B^2$, where B is the intensity of the magnetic field. Thus, for values of B applicable to the geomagnetic field connected with the auroral zone of, say 4×10^{-3} gauss, $\gamma \sim 10^3$. It is apparent from (7) that $\frac{\partial\rho}{\partial t}$ would be nearly zero on the interior of a plasma density feature where n becomes significantly different from zero. Taking a single plasma density distribution of the form $n(y) = a \sin^2 (\frac{\pi}{L})y$, where a is the maximum n and L is the width of the distribution, leads to the result that the maximum of $\frac{\partial\rho}{\partial t}$ occurs within a distance of about $2L/\gamma_{max}$ on the edge of the distribution, where $\gamma_{max} = 4\pi mc^2\ a/B^2$. For $b \sim 4 \times 10^{-3}$ gauss and $n \sim 10$ particles/cm^3, the maximum of $\frac{\partial\rho}{\partial t}$ would occur within a distance of about $L/10^3$ from the edge of the distribution. The regions of excess charge are therefore very thin and appear very near minima in the neutral plasma density for the model analyzed here. The same conclusion can be drawn for the electron excess corresponding to the proton excess dealt with here. The constant γ derived from the proton motion must also be applied to the formation of an electron charge excess, since polarization drift of protons will affect the formation of an electron sheet.

In order to fully illustrate this result, the thin-sheet character of the excess-charge sheath can be derived from consideration of the electric field. The excess charge is the source of the electric field. Hence, 0.99 of the excess-charge must appear between the edge of the distribution and a point inside the distribution where the time rate of change of E is 0.99 the maximum $\frac{\partial E}{\partial t}$. For the sample plasma number density distribution $n(y) = a \sin^2 \frac{\pi}{L} y$ considered here, a sheath thickness b can be defined as that distance inside the distribution which includes 0.99 of the excess charge. Then from (7), $(\frac{\partial E}{\partial t})_{max}$ occurs for $n(\frac{L}{2}) = a$. For $\frac{\partial E}{\partial t} = 0.99 (\frac{\partial E}{\partial t})_{max}$, $1 - e^{-\gamma n(b)} = 0.99 (1 - e^{-\gamma a})$. In general, for $a > 10^{-2}$ particles/cm3, $e^{-\gamma a} \ll 1$. Hence $e^{-\gamma n} = 1 - 0.99 = 0.01$, and $n(b) = (1/\gamma) \ln 100$. Since $\gamma \sim 10^3$, $n(b) = 4.6 \times 10^{-3}$ particles/cm^3. For small n(b)/a, $\sin \pi b/L \sim \pi b/L = \sqrt{n(b)/a}$, hence $(b/L) = (1/\pi) \sqrt{n(b)/a}$. For a = 10 particles/cm^3, this gives $b/L \simeq 7 \times 10^{-3}$. Table 1 gives a resume of b/L (as defined here for the plasma density distribution $n(y) = a \sin^2 (\pi/L) y$) for different values of the maximum number density a. Also shown is the thickness b in km for L = 1000 km. The values of b/L shown here would be considered applicable to the equatorial plane. Projected along $\underset{\sim}{B}$ into the ionosphere in the auroral zone, sheets of thickness b would be reduced in thickness by a factor of about 30. Thus a trapped plasma feature 1000 km in thickness with a peak density of 1 particle/cm^3 in the equatorial plane might lead to an auroral arc with a thickness of 2/3 km. The incident particle flux in such an aurora might typically be 10^7 particles/cm^2 sec. This would require a mean rate of charge separation of 10^7 charged particles/sec in a magnetic

Table 1

RESUME OF b/L

Ratio b/L of Excess Charge Sheath Thickness b to Plasma Density Perturbation Thickness L for Plasma Density $n(y) = a \sin^2 (\pi/L)y$, Where a is Peak Plasma Density

Peak Density a Particles/cm^3	Ratio of Sheath Thickness to Perturbation Thickness b/L	Sheath Thickness b for L = 1000 km Plasma Density Perturbation, km
0.01	0.22	220
0.1	0.068	68
1	0.022	22
10	0.007	7
100	0.002	2

flux tube (connected with the auroral zone) that has a volume of about 10^{12} cm^3. From Eq. (8), such a flux could be supplied by protons drifting with a mean velocity $\bar{v}_B$ of about 100 m/sec and distributed so that gradients $\frac{dn}{dy}$ occur of the order a/L or 10 particles/cm^3/1000 km, provided $\frac{dn}{dy}$ is finite very close to the zeros in n(y).

A background plasma would suppress $\frac{\partial\rho}{\partial t}$, as can be seen from (8). Consider $n(y) = n_o + n_p(y)$, where n_o is constant and $n_p(y)$ is the perturbation plasma density. If $n_o > 0$, (8) can be written $\frac{\partial\rho}{\partial t} = e\bar{v}_B\, e^{-\gamma n_o}\, e^{-\gamma n_p}\, \frac{dn}{dy}$. This rate of change of ρ is obviously less than that which would hold for n_p alone by the factor $e^{-\gamma n_o}$ resulting from the background plasma. This effect will serve to reduce the charge separation and hence any resulting auroral particle fluxes. It can therefore be concluded that for auroral displays to be a result of charge separation in the magnetosphere, the background plasma must be discontinuous. If structure in the plasma density in the magnetosphere is due to plasma instabilities, the background plasma is likely to have a structure at least as complex as that inferred for higher energy particles (Rosen, Farley, and Sonett, 1960).

One inference can be drawn from this admittedly oversimplified treatment of charge separation in a two-dimensional plasma. That is, if the electric fields resulting from charge separation are increasing, there will be a tendency for excess charge to appear in fairly restricted regions near the outer boundaries of plasma density perturbations. Thus it seems possible to account for the extreme thinness of auroral sheets within the framework of an auroral theory based on charge separation resulting from adiabatic particle motion.

It will be seen in Section III that increasing electric fields are also required to account for continuing discharge of auroral particles from a geomagnetically trapped plasma.

III. THE LOWERING OF THE MIRROR POINTS OF TRAPPED PARTICLES

When charges separate, as described above, reflection of the trapped particles occurs at lower altitudes, provided that electric fields arise parallel to the magnetic field lines. As will be suggested in Section IV, electrojets may result when thermal particles neutralize excess charge of opposite sign, but fluxes of more energetic particles into the atmosphere are required to form auroral displays. As pointed out by Kern and Vestine (1961) and Chamberlain (1961), many of the features of auroral morphology can be accounted for, if excess charge distributions can be maintained in the trapped radiation.

Chamberlain (1961) discusses the lowering of mirror points in regions of excess charge. The drift of particles antiparallel to the direction of the transverse electric force on particles leads to the conversion of transverse kinetic energy W_1 into electrostatic potential energy at the rate $\dot{W}_1 = - e\underset{\sim}{E} \cdot \underset{\sim}{v}_d = - e\dot{V}$, where $e\underset{\sim}{E}$ is the electric force on the particle, $\underset{\sim}{v}_d$ is its drift velocity, and where $e\dot{V}$ is the rate of change of electrostatic potential energy of the particle. Thus the pitch angle of a particle, relative to the magnetic field, changes as its electrostatic potential changes. Discharge of a particle from a region of high electrostatic potential (a region of excess charge) along magnetic field lines leads to a recovery of this kinetic energy. The system considered by Chamberlain is a flux tube, containing excess charge, of about the same dimensions as the Larmor radius of the trapped particles. In this configuration, the transverse electric field of the excess charge varies in direction during a single orbital cycle. The magnetic moments of particles are not invariant for an electric field of this geometry, although $\frac{W_\perp}{B}$ still is.

For the two-dimensional geometry considered here (Fig. 2), the transverse electric field is essentially constant in direction and magnitude during a single orbital cycle. In this case, although the transverse kinetic energy changes while the particles drift, the particles' magnetic moment is invariant (Hellwig, 1955). In fact, it can be shown that the time rate of change of the transverse kinetic energy W_1 is given by $\dot{W}_1 = \mu\dot{B}$, where μ is the particle's magnetic moment, and $\dot{B}$ the time rate of change of B due to $\underset{\sim}{E}\times\underset{\sim}{B}$ drift motion in a nonuniform magnetic field $\underset{\sim}{B}$. This is equivalent to a betatron deceleration of the transverse motion.

In regions of excess charge, accelerations of particle motion parallel to $\underset{\sim}{B}$ may occur. In order to relate auroral displays to particle fluxes along $\underset{\sim}{B}$, a model electric field will be defined. This electric field is assumed to arise from excess charge resulting from the charge separation mechanism as described in Section II. The effects of such an electric field on a trapped plasma will be examined.

The effect of a model electric field component E_2 parallel to the magnetic field $\underset{\sim}{B}$ can be examined using the equations of motion for a single particle. In general, there will also be a transverse component E_1; this will be neglected here. The acceleration of a particle $\ddot{z}$ parallel to $\underset{\sim}{B}$ will be given by

$$\ddot{z} = (1/m)(-\mu\,\nabla_2 B + eE_2) \tag{9}$$

where m is the mass of the particle, μ the magnetic moment, $\nabla_2 B$ the component of the gradient of $\underset{\sim}{B}$ parallel to $\underset{\sim}{B}$, e the charge of the particle, and E_2 the component of the electric field parallel to $\underset{\sim}{B}$. Here z is taken as a displacement from the equatorial plane along $\underset{\sim}{B}$, and the electric force eE_2 is taken in the same direction. Particles of opposite charge can be treated by taking the opposite sign for e, along with the appropriate mass and magnetic moment. Initiation of charge separation will lead to the discharge along $\underset{\sim}{B}$ of the particles with the smallest magnetic moments from the region of excess charge. As such particles discharge, those remaining will have larger moments. At any given time, it seems plausible to define a lower bound μ_ℓ for the moments of the remaining particles. The magnitude of this smallest magnetic moment μ_ℓ will evidently increase as discharge continues.

The field required to permit free motion of particles along $\underset{\sim}{B}$ is given by equating $\ddot{z}$ to 0 in (8) for particles with this smallest moment μ_ℓ. This is just

$$E_2 = (\mu_\ell/e)\,\nabla_2 B \tag{10}$$

This electric field will produce a flux Φ of particles with magnetic moments μ_ℓ into the atmosphere. The flux Φ must correspond to the total drift flux of excess charge into the sides of the region.

The latitude oscillation time for a particle with magnetic moment μ_ℓ can be computed for a given magnetic field line and mirror field B_m in the absence of electric fields. The acceleration of such a particle travelling from the mirror point to the equatorial plane is given by $\mu_\ell\,\nabla_2 B/m$, where $\nabla_2 B$ is the parallel component of the magnetic field gradient and m is the particle mass. Consider now a thermal particle of opposite sign travelling from the same mirror point to the equatorial plane in the model electric field E_2. The acceleration of this particle will be given by eE_2/M, where e is the charge and M the mass of the thermal particle, and the model electric field E_2 is given by (10). It is evident that a thermal particle of opposite charge undergoes an acceleration m/M times that undergone by a particle with moment μ_ℓ mirroring in the atmosphere in the absence of electric fields. Hence such a thermal particle has a travel time from the atmosphere to the equatorial plane which is $\sqrt{M/m}$ times the travel time of a particle of opposite sign oscillating between mirror points in the absence of an electric field.

A model electric field sufficient to cause 6 Kev electrons with moment $\mu = 2 \times 10^{-8}$ emu to discharge along $\underset{\sim}{B}$ into the atmosphere will therefore accelerate atmospheric protons to the equatorial plane in $\sqrt{1840}\,\tau$, where τ is the travel time of an electron with moment $\mu = 2 \times 10^{-8}$ emu which mirrors in the atmosphere in the absence of an electric field. This time τ is of the order of 1 sec. Time scales for proton neutralization of electron charge excesses are therefore large, allowing the development of electron excesses that can discharge to generate auroral displays. Electrons from the atmosphere can quickly neutralize proton-charge excesses. Chamberlain (1961) estimates lifetimes of the order of 1 sec for proton charge excesses.

The above argument indicates that transient discharges of electrons can be expected. Auroral displays, however, can persist for many minutes, and in some cases hours. It is therefore important to estimate possible ion fluxes upward along $\underset{\sim}{B}$ into an auroral flux tube. The model electric field E_2 will lead to such upward motion of ions. In the conducting ionosphere, the ion current density along $\underset{\sim}{B}$ can be represented by $j_i = \sigma_{oi}E_2 = (N_i e^2/m_i\nu_i)E_2$, where $\sigma_{oi} = N_i e^2/m_i\nu_i$ is the ion conductivity along $\underset{\sim}{B}$, N_i is the local number density, e is the charge, m_i is the mass, and ν_i is the collision frequency of the ions. If the ions considered are protons, the charge is the electron charge. Now the net ion flux upward is limited by the conductivity along $\underset{\sim}{B}$ near the bottom of an auroral flux tube. Ion motions in the outer parts of such a flux tube will act to redistribute excess charge resulting from charge separation. Charge neutralization, however, will occur only by the introduction of a net ion flux into the tube. Except for the regions in the ionosphere where the conductivity transverse to $\underset{\sim}{B}$ is significant, a net ion flux can only be introduced into an auroral flux tube by conduction along $\underset{\sim}{B}$. The value of the net ion flux into an auroral flux tube can be obtained if the electric field E_2 along $\underset{\sim}{B}$ is specified at, say, the F_2 maximum in the ionosphere. If this ion flux is less than the rate at which excess charge is appearing in the flux tube, the electric field can increase and energetic electrons can be discharged. The ion conductivity σ_{oi} along $\underset{\sim}{B}$ in the F_2 region of the ionosphere at an altitude of 300 km is of the order of 10^{-12} emu. The electric field E_2 that will discharge 6-Kev electrons to this altitude is about 2×10^3 emu. For this electric field, the ion current density is therefore $10^{-12} \cdot 2 \times 10^3$ abamp/cm^2 = 2×10^{-9} abamp/cm^2. This corresponds to an ion flux density of 10^{11} protons/cm^2 sec upward along $\underset{\sim}{B}$. This net upward ion flux is very nearly the same as the flux density of 6-Kev electrons observed by McIlwain (1960) in a bright auroral arc. Hence, charge separation rates inferred from auroral electron fluxes can be great enough to equal or exceed ion charge transport upward along $\underset{\sim}{B}$. If this is the case, the electric field E_2 along $\underset{\sim}{B}$ can increase with time and continuous discharge of trapped electrons is possible.

A fictitious magnetic moment μ' can be defined for a particle. Let $\mu' = \mu \pm \mu_\ell$, where μ is the actual particle moment (which is invariant), μ_ℓ is as defined earlier. The plus sign is taken for particles with the opposite charge of the excess in the region, the minus sign is taken for particles with the same charge as that of the excess. Then, the acceleration parallel to $\underset{\sim}{B}$ is given by $\ddot{z} = -(1/m)\, \mu' \nabla_2 B$. It follows that $\ddot{z} = -(1/m)(\mu \pm \mu_\ell) \nabla_2 B$. This is the equation of motion for particles moving in the combined electric and magnetic fields. Hence, particle motion is modified by the electric field E_2 in the same way as if the particle's magnetic moment were changed and no electric field were present. For electrons moving in a flux tube containing an electron excess, the fictitious magnetic moment of an electron is $\mu' = \mu - \mu_\ell$, where μ_ℓ is the lowest electron moment present, and μ is the real magnetic moment. Note that for a constant electric field, μ_ℓ is constant, and hence μ' is invariant.

Changes in the mirror fields of particles can be examined using the fictitious magnetic moment. The kinetic energy of a particle as it crosses the equatorial plane is modified by transverse drift in the nonuniform electric and magnetic fields. If the magnetic moment μ is constant, the kinetic energy at a new equatorial magnetic field B_o' can be given by $W_o' = W_o - \mu(B_o - B_o')$, where W_o is the particle's kinetic energy prior to charge separation, and B_o is the initial value of the magnetic field where the particle crosses the equatorial plane. Applying the idea of the fictitious magnetic moment, note that the new mirror field B'_m in the presence of the electric field E_2 is given by W'_o/μ'. Also note that $W_o = \mu B_m$, where B_m is the original mirror field of the particle. Thus $B'_m = (\mu/\mu')(B_m - B_o + B'_o)$, where all of the quantities have been previously defined. It is evident that if B'_o and B_o are specified, the new mirror field B'_m can be calculated. For the case in which $B_o - B'_o$ approaches zero, or is, at least, small compared to B_m,

$$B'_m \cong (\mu/\mu')\, B_m \tag{11}$$

Since $\mu' = \mu \pm \mu_\ell$, the new mirror field is given by $B'_m = (1 \pm \mu_\ell/\mu)^{-1}\, B_m$, where the minus applies to particles with the same charge as the excess charge giving rise to the electric field along $\underset{\sim}{B}$, and the plus applies to particles of opposite charge.

The mirror point energies of particles can be computed using the invariance of the real magnetic moment μ. The kinetic energy of a particle at the new mirror field B'_m is given by $W = \mu B'_m$. This is just $(B'_m/B_m)\, W_o$, where W_o is the original kinetic energy, and B_m is the original mirror field.

The auroral flux produced by an electric field E_2, as defined above, can be computed. The time constants for particle discharge to the atmosphere can thus be evaluated. Consider a given initial distribution of trapped-particle energies within a magnetic flux tube linking the auroral zone.

In the following treatment, it will be assumed that the particles trapped within a given magnetic flux tube are not resupplied by adiabatic drift. The source of electrostatic energy is assumed to be an excess of trapped particles of one charge appearing in the flux tube as a result of charge separation. An auroral flux will thus be due to discharge of particles initially contained in the flux tube. This can be seen to occur for the case in which more energetic particles of one kind (say, protons) drift out of a flux tube, leaving an excess of particles of opposite charge.

The polarization component of adiabatic drift motion which results from increasing electric fields transverse to $\underset{\sim}{B}$ described in Section II would inhibit the drift of low energy particles into a region of separated charge. Thus it might be expected that any drift supply of particles to regions of excess charge would be mainly higher energy particles. If this is the case, the process described here would give the auroral flux from a background plasma in a region of separating charge.

For an isotropic distribution of particle velocities at the equatorial plane, the differential particle-density distribution can be given by

$$dn = n\ g(W)\ dW\ d\cos\alpha \tag{12}$$

where $g(W)$ is the distribution function for particle kinetic-energies, α is the pitch angle and W the kinetic-energy for a given particle, and n is the total particle density. Now $W = \mu B_m$, and $dW = B_m\, d\mu$, for a given pitch angle α, where μ is the particle magnetic moment and B_m is the mirror field. Also $\sin^2\alpha = B_o/B_m$, where B_o is the equatorial field. Hence, $d\cos\alpha = \frac{1}{2}\,(B_o/B_m^2)(1 - B_o/B_m)^{-1/2}\, dB_m$. It follows from (12) that

$$dn = (n/2)\ g(\mu B_m)(B_o/B_m)(1 - B_o/B_m)^{-1/2}\ dB_m\ d\mu \tag{13}$$

The total number of particles initially in an auroral flux tube with an area of 1 cm^2 in the auroral ionosphere will be defined here as $N \equiv Vn$ where n is the particle density referred to above and V is the "effective volume" of the flux tube. For a magnetic flux tube connected with the auroral zone $V \cong 10^{12}$ cm^3 between the ionosphere and the equatorial plane. Now from (13) and the definition of N,

$$dN = N\ f(\mu, B_m)\ dB_m\ d\mu \tag{14}$$

where

$$f(\mu, B_m) = (B_o/2)\left[g(\mu B_m)/B_m{}^{1/2}\ (B_m - B_o)^{1/2}\right]$$

Thus the particles within the flux tube can be described in terms of a distribution of magnetic moments and mirror fields.

The value of the magnetic field in the auroral ionosphere B_a can be taken as a lower limit for trapping. If an electric field E_2 exists along B such that the mirror field for a particle becomes greater than B_a, the particle can be said to be discharged into the atmosphere. That is, from Eq. (11) if initially the mirror field B_m is greater than

$$B_m = (1 - \mu_\ell/\mu)\ B_a = (\mu'/\mu)\ B_a$$

for a given μ_ℓ defining E_2 by (10), the particle is discharged. The number of particles still trapped at a given value of E_2 is thus given by

$$N' = N\int_{\mu_\ell}^{\infty}\int_{B_o}^{B_a\ \mu^1/\mu} f(\mu, B_m)\ d\mu\ (\mu'/\mu)\ dB_m' \tag{15}$$

The time rate of change of N' exactly corresponds to the auroral flux Φ. Thus differentiating (15), it can be shown that

$$\Phi = -\dot{N}' = -N\,\dot{\mu}_\ell\,B_a \int_{\mu_\ell}^{\infty} \frac{d\mu}{\mu}\; f(\mu, \frac{\mu'}{\mu}\; B_a) \qquad (16)$$

where $\dot{\mu}_\ell$ is the rate of change of the magnetic moment μ_ℓ that defines E_2. Now the smallest magnetic moment remaining in the flux tube is given by $\mu_1 = \mu_\ell\, B_a/B_a - B_o)$, since the equatorial magnetic field is not zero. This moment defines the lower limit for the integral in Eq. (16). The integrand in (16) is determined from Eq. (14). Suitable change of variables leads to

$$\Phi = N\,\dot{\mu}_\ell\,B_o \int_{x_1}^{\infty} \frac{g(x)\;dx}{2x^{1/2}\;(bx - a)^{1/2}} \qquad (17)$$

where $x = (\mu - \mu_\ell)\,B_a$, $x_1 = \mu_\ell B_o/(1 - B_o/B_a)$, $b = (1 - B_o/B_a)$, and $a = \mu_\ell B_o$.

Now g(x) can be given for a kinetic-energy spectrum of the form W^{-k} where k is an integer. In this case

$$g(x) = 2(k - 1)\; W_{Min}^{k-1}\; W^{-k}$$

where W_{Min} corresponds to x_1 and is given by $W_{Min} = a/b$ as defined above. W_{Min} is the smallest particle kinetic-energy in the distribution. Hence,

$$\Phi = N\dot{\mu}_\ell\;B_o\;(k - 1)\;(\tfrac{a}{b})^{k-1} \int_{x_1}^{\infty} \frac{dx}{x^{k+1/2}\;(bx - a)^{1/2}} \qquad (18)$$

and on integration,

$$\Phi = N(1 - B_o/B_a)^{1/2} K(k) \frac{\dot{\mu}_\ell}{\mu_\ell} \tag{19}$$

where K(k) is given by

$$K(k) = \frac{(2k - 2)(2k - 4)\ldots(2)}{(2k - 1)(2k - 3)\ldots(3)} (k - 1) \tag{20}$$

For k = 5, K(k) = 1024/315, while for k = 2, K(k) = 2. The energy spectrum is evidently not an overwhelming factor in determining the auroral particle flux produced by a given particle density, electric field, and time rate of change of electric field.

In order to determine the time scale of an auroral flux and of the electric field E_2 along $\underset{\sim}{B}$, Eq. (19) can be written

$$\dot{N} = - N\beta \, \dot{\mu}_\ell/\mu_\ell \tag{21}$$

where $\beta = K(k) \, (1 - B_o/B_a)^{1/2}$. Thus, integrating (21), the integral of the flux Φ over time for a given display can be derived as

$$\int_0^t \Phi \, dt = N_o - N = \left[1 - (\mu_\ell/\mu_{\ell_o})^{-\beta}\right] N_o \tag{22}$$

where N_o is the initial number of particles in the flux tube, N is the number at time t, μ_{ℓ_o} is the initial smallest moment, μ_ℓ is the smallest moment at time t, and β is as given above. Therefore, the time required for an auroral flux of constant magnitude to deplete a region of particles with moments less than 100 μ_{ℓ_o} will be about N_o/Φ. If $N = 10^{12}$ cm^3 · 10 particles/cm^3 and $\Phi = 10^9$ particles/cm^2 sec, then $t = 10^4$ sec, or about 3 hours.

The flux of atmospheric particles up magnetic field lines is neglected in the above calculation of auroral flux. This calculation could be considered applicable to electron excesses in the trapped plasma. For electron excesses, the flux of electrons down field lines would not necessarily exceed the flux of atmospheric protons up field lines. The electron flux would, however, contribute the energy associated with the visible aurora. Hence auroral electron flux estimates based on calculations from the spectra of aurora, or measured directly in the ionosphere, must be taken as implying lower limits to the rate at which excess charge is supplied to a given magnetic flux tube. For a particle flux of 10^7 particles/cm^2 sec in the auroral zone, the lower limit to the rate at which excess charge is supplied near the equatorial plane (in, say, 1/10 the volume of the flux tube) would be about 10^7 particles/10^{11} cm^3 sec or 10^{-4} particles/cm^3 sec. This rate of charge separation could be obtained with drift velocities of the order of 100 m/sec and plasma density gradients of the order of 10^{-3} (particle/cm^3)/km. If the process of formation of thin auroral sheets proposed in Section II is effective, the auroral flux would be restricted to localized regions connected along $\underset{\sim}{B}$ to the boundaries of irregularities in the trapped plasma.

A further result of the electric-field model for discharge of trapped particles is that some variation of the energies of auroral particles as a function of time would be anticipated. Solving Eq. (22) for the moment μ_ℓ gives

$$\mu_\ell = \mu_{\ell_0} \left[1 - (1/N_0) \int_0^t \Phi \, dt \right]^{-1/\beta} \tag{23}$$

Now the moment μ_ℓ is given very nearly by W/B_a, where W is the discharged energy of a particle and B_a is the atmospheric magnetic field. Similarly $\mu_{\ell_o} = W_o/B_a$ where W_o is the initial energy of a discharged particle at time t = 0. Note that W_o is not necessarily zero. Thus at time t, the energy of a particle mirroring in the atmosphere is given by

$$W = W_o \left[1 - (1/N_o) \int_0^t \Phi \, dt \right]^{-1/\beta} \tag{24}$$

The quantity in brackets goes to zero as t becomes large. Thus, as particles of lower energy are discharged and as the component of the electric field parallel to $\underset{\sim}{B}$ increases with time, the energies of particles producing a given auroral display increase with time.

IV. POLAR-ELECTROJET CURRENT SYSTEMS

As mentioned in Section I, charge separation in the geomagnetically trapped plasma has been invoked to account for the occurrence of polar-electrojet current systems (Vestine, 1960; Chamberlain, Kern, and Vestine, 1960; Kern and Vestine, 1961; Kern, 1961). The process of charge separation described in Section II transports charge across plasma density irregularities in the trapped radiation. The electric fields and potentials associated with separated charge are short-circuited by conduction along magnetic field lines into the atmosphere. Thus, atmospheric current systems would be associated with such charge transport. Allowing magnetic field lines to be equipotential lines does not change this situation. A finite resistance to particle flow in the atmosphere ensures differences of potential between regions of excess positive and negative charge in the trapped radiation. The system can be visualized as involving a specified current applied to a medium of anisotropic conductivity; i.e., the atmosphere.

If polar-electrojet current systems are ascribed to Hall conduction caused by meridional electric fields in the E-region (Baker and Martyn, 1953; Chamberlain, Kern and Vestine, 1960; Akasofu, private communication), eastward-directed electrojets are associated with poleward-directed electric fields and westward-directed electrojets with equatorward-directed electric fields. The drift directions produced by eastward-directed magnetic-field gradients are such as to produce poleward-directed electric fields, while westward-directed magnetic gradients produce equatorward-directed electric fields in the trapped radiation incident in auroral regions. Eastward-directed electrojets may therefore be associated with eastward-directed magnetic-field gradients, and westward-directed electrojets with westward-directed gradients (as in Fig. 3).

The extent in longitude of such meridional electric fields would correspond to the extent of the eastward- or westward-directed magnetic-field gradients where irregularities in plasma density exist to allow the development of polarization. Electrojets would develop where sufficient trapped radiation, polarized and discharged by charge separation, penetrates the ionosphere. A hypothetical distribution of eastward and westward components of the geomagnetic-field gradient in a shell of trapped particles is shown in Fig. 4. Adiabatic drift of electrons before midnight in this model is northward to higher-altitude mirror points for a given mirror-point field; after midnight, the electron mirror points drift south to lower altitudes. The increase of ionospheric conductivity encountered by the electrons on penetration to lower altitude would seem to imply that polar electrojets due to Hall conduction will have greater intensity after midnight than for a positive bay current system developing before midnight.

The process described above requires that the direct conduction current associated with the Hall-current electrojets be supplied by continuous charge separation in the trapped radiation. This current is equal therefore to the charge transport, $nev_d r\ell$ per radian of longitude, where n is the plasma number density, $e = 1.6 \times 10^{-20}$ emu is the electronic charge, v_d is the separation velocity for particles of opposite sign, $r = 5 \times 10^9$ cm is the average radius of the region of drift separation conjugate to auroral latitudes, and $\ell = 2 \times 10^9$ cm is the approximate extent of the drift region along lines of force. The total direct conduction current per radian of longitude in the auroral region associated with a typical electrojet can be estimated. If Baker and Martyn's (1953) value for the height-integrated direct conductivity of 10^{-8} emu and an estimated electric field of 10^4 emu are used, and if the radius of the auroral zone is taken as 2.5×10^8 cm, this current turns out to be 2.5×10^4 emu per radian of longitude in the auroral zone. Equating this to the drift current given above, and taking $n = 10$ particles/cm^3, permits calculation of the drift separation velocity required to maintain such a current as $v_d = 1.5 \times 10^4$ cm/sec.

We can apply the equation given earlier for the drift velocity to obtain an estimate of the required geomagnetic field distortion. Since the magnitude of the east-west component of the magnetic field gradient can be written $\nabla_\Phi B = (1/r)\, dB/d\Phi$, from (1) $dB/d\Phi = B^2 e r v_d W(1 + \cos^2 \alpha)$. With $B = 10^{-3}$ gauss at 7 to 8 earth radii, $e = 1.6 \times 10^{-20}$ emu, $r = 5 \times 10^9$ cm, $v_d = 1.5 \times 10^4$ cm/sec, $W = 6$ Kev, and taking the average value for α of $\pi/4$, we obtain $dB/d\Phi \sim 10^{-4}$ gauss/radian. This amount of distortion would be considered applicable to times of magnetic bays. Note here that the energy density of the particles required in the trapped plasma is of the order of 60 Kev/cm^3. This is about 1/3 the energy density of the geomagnetic field at 7 earth radii. If the energy density of the particles is higher, smaller distortions could lead to the necessary charge transport. When the energy densities are less, on the other hand, the distortions of the field must be greater in order to transport sufficient charge to drive the electrojets.

V. THE POLAR AURORA

The adiabatic charge-separation mechanism described in Section II, together with the discharge mechanism discussed in Section III, provides a means for producing the polar auroras from a plasma trapped in the geomagnetic field. The results indicate that auroral-particle fluxes will depend on (1) the energy density of particles in the plasma, (2) the distortion of the geomagnetic field, and (3) plasma density gradients associated with irregularities in the trapped plasma. The energies of incident auroral electrons are found to increase as a function of time because of the dependence of auroral-particle flux on the distribution of particle energies and magnetic moments within the plasma connected along $\underset{\sim}{B}$ with the auroral region.

In Section II, it was pointed out that the formation of thin auroral sheets might be a natural consequence of the proposed charge separation. This geometry has been considered by Kern and Vestine (1961), who point out some of the consequences of charge separation. Among these consequences is the fact that sheet beams of charged particles are subject to electrodynamic instabilities as demonstrated both experimentally and theoretically (Kyhl and Webster, 1956; Pierce, 1956; Webster, 1955, 1957). Such considerations can be applied to auroral morphology to account for the observed sequences of auroral forms in terms of changes in the velocity and flux density of incident auroral particles.

The principles discussed above can be used to interpret observations of auroras and electrojet current systems. For example, one form of distortion of the geomagnetic field might be such as to produce components of the geomagnetic field gradient directed toward the midnight meridian in the equatorial plane on the night side of the earth. Such gradients can be thought of as lying in surfaces of constant plasma density in the magnetosphere. If this is the case, charge separation of the kind discussed in Section II will lead to polarization of the trapped plasma. The pattern described here is shown in Fig. 4. If the region of trapped plasma is limited in extent, prior to midnight an electron excess will accumulate on the outer boundary. After midnight, a proton excess will appear on the outer boundary.

The systems of currents and the meridional electric fields associated with charge separation of the kind shown in Fig. 4 are such as to account for many features of the magnetic observations of Heppner (1954) for College, Alaska. Observed drift of auroral structures are accounted for in terms of $\mathbf{E} \times \mathbf{B}$ motor drift, westward drift being produced before midnight and eastward after midnight (Nichols, 1957; Kim and Currie, 1958). The drift of incident protons southward before and northward after midnight, as observed by Reid and Rees (1961), is also accounted for by this configuration.

Auroral morphology and drift directions of Davis (1962) are shown in Fig. 5. These distributions of auroral forms imply an east—west charge separation derived from Störmer-orbit motion in the geomagnetic field combined with radial drift separation due to eastward or westward components of the geomagnetic-field gradient shown in Fig. 4. The fixed spatial relation of the auroral pattern with respect to the sun observed by Davis (1962) is of course basic to the idea that charge separation is caused by interaction of the magnetosphere and an ionized solar stream.

Heppner (1954) shows that auroral forms coinciding with peak westward-directed electrojets are often active rayed forms. Such forms would be consistent with the model developed above, in that they would coincide with peak electrostatic-field development in the trapped radiation. The activity would be ascribed to electrodynamic instability arising from the characteristic behavior of beams of energetic charged particles (Webster, 1957; Kern and Vestine, 1961). The peak electrojet current would be similarly associated with the highest electric field in the trapped radiation.

Irregularities in plasma density are required to generate thin auroral structures. The assumption that such irregularities exist in the neutral plasma connected with auroral regions is supported by observed fluctuations in the counting rates of satellites and probes passing through this region, which have been interpreted to indicate just such structures (Rosen, Farley, and Sonett, 1960). Direct evidence for the "dumping" of trapped radiation has been obtained through correlation of Explorer VII (1959 iota) radiation measurements with observations of visible and subvisible auroral emissions (O'Brien, et al., 1960). Additional evidence for the dumping of trapped radiation has been obtained from an analysis of satellite observations by Rothwell and McIlwain (1960).

Electric fields parallel to $\underset{\sim}{B}$ have been invoked by McDiarmid, Rose, and Budzinski (1961) to interpret their rocket measurements of pitch angle and energy distributions in the primary electron flux in an aurora. Their results also indicate space and time separation of electron and proton fluxes. This would support the charge-separation hypothesis.

Observations of sunlit auroras at high altitudes give additional evidence for the existence of electric fields parallel to $\underset{\sim}{B}$. The presence in such auroras of N_2^+ ions with number densities of the order of $10^2/cm^3$ is inferred from the fact that they are bright enough to be seen. Bates (1960) suggests that such ions must propagate upward, since they are not normally present in these regions in such large numbers. Upward motion of positive ions is to be expected in auroral structures produced by the electric field mechanism described earlier.

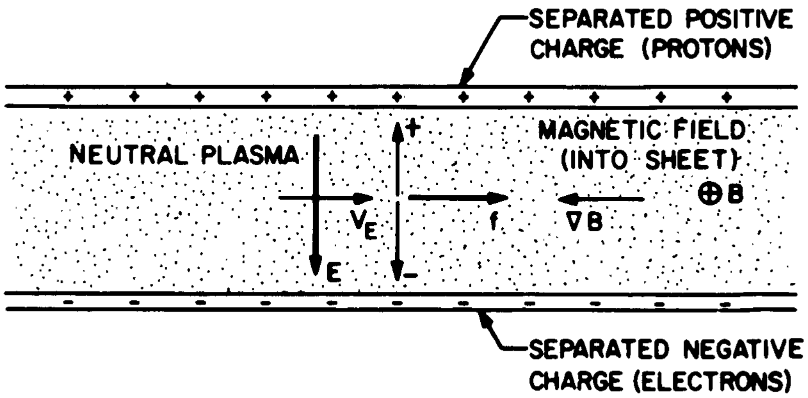
Horseshoe auroral arcs can be explained as a consequence of charge separation occurring in irregularities in the trapped neutral plasma which are limited in east — west extent. Numerous so-called "horseshoe aurora" have been reported. Störmer (1955) shows two illustrations of this auroral form. Analyses of auroral observations have indicated that this auroral form is not rare, but is somewhat difficult to photograph. Observations available from probe and satellite counting rates indicate that irregularities in the plasma number density may indeed exist (Rosen, Farley, and Sonett, 1960). Given such irregularities, it can be seen that adiabatic east — west particle drift will lead to charge separation. This situation is represented in an equatorial section in Fig. 6; the charge separation depicted leads to a charge-density distribution that is horseshoe-shaped. Increasing electric fields will lead to precipitation in a horseshoe-shaped arc in the auroral zone. If the mechanism for auroral sheet formation based on polarization drift is effective, such arcs can be very thin compared to the plasma irregularities (see Section II).

VI. RECURRENCE OF POLAR AURORA AND ELECTROJETS

It has been shown that the polar aurora and electrojets may arise as a product of adiabatic particle motion in a geomagnetic field distorted by a solar stream. Recurrence of electrojets and aurora at about the same longitude on successive nights can be expected if there are low-energy particles in the trapped radiation contiguous to the auroral regions. Low-energy particles drift very slowly in longitude with respect to the earth's surface, and would effectively rotate with the geomagnetic field. Such a low-energy plasma would exhibit the pattern of adiabatic drift separation of charge imposed by solar-stream distortion of the geomagnetic field. Local sequences of polar aurora and electrojet phenomena can therefore be expected to reflect a 24-hour period corresponding to the rotation period of the geomagnetic field.

Such distortion has not yet been completely treated analytically. It would seem that the atmospheric phenomena of polar aurora and electrojets may provide some information regarding the nature of the solar-stream geomagnetic-field interaction. For example, the hypothetical east—west geomagnetic-field gradients proposed (see Fig. 4) to account for the auroral morphology patterns of Davis (1962) indicate that solar streams may increase the geomagnetic field near midnight. Such patterns may also be accounted for by the motion of field-line shells with respect to surfaces of constant field.

If curvature of field lines is considered, the pattern proposed here would correspond to the warping of field lines out of meridional planes toward the midnight meridian. Such warping might result if solar streams tend to extend lines of force on the night side of the magnetosphere.



∇B: MAGNETIC FIELD GRADIENT, TRANSVERSE TO FIELD GIVING RISE TO TRANSVERSE FORCE PER PARTICLE $f = \mu \nabla B$

E: ELECTRIC FIELD OF SEPARATED POSITIVE AND NEGATIVE CHARGE

V_E: NEUTRAL $\underline{E} \times \underline{B}$ MOTOR DRIFT VELOCITY DUE TO ELECTRIC FIELD E

Fig. 1 — Charge separation in a bounded plasma due to transverse geomagnetic field gradient

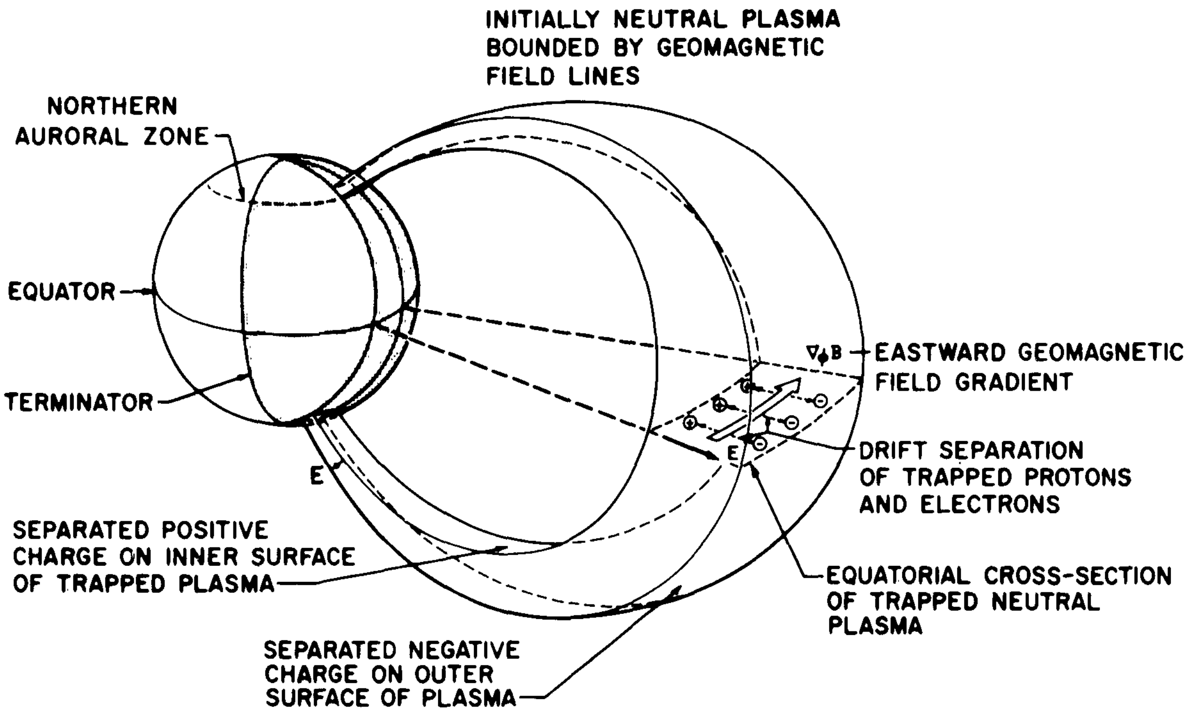


Fig. 2 — Charge separation in geomagnetically trapped radiation due to eastward geomagnetic field gradient

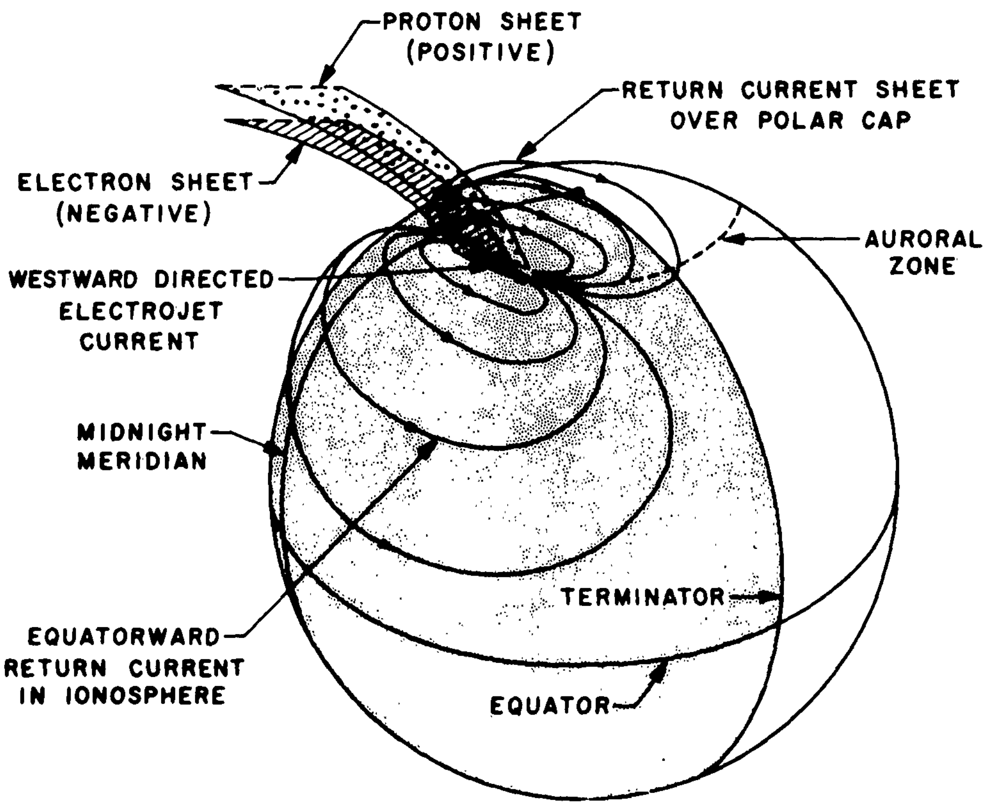


Fig. 3 — Polarization of radiation incident in the auroral zone and the Hall conduction polar electrojet currents

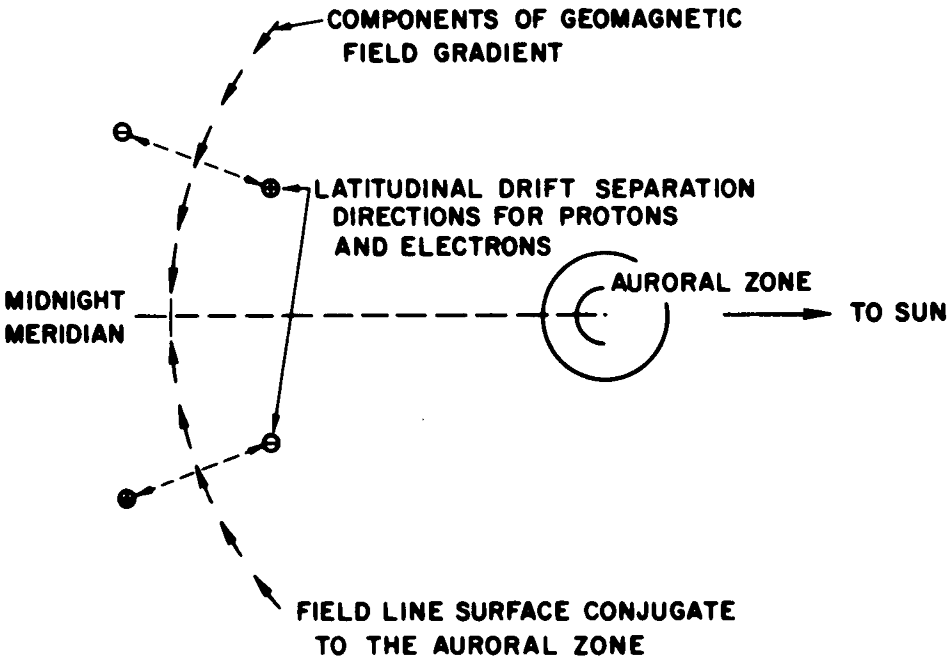


Fig. 4 — Hypothetical eastward and westward components of geomagnetic field gradient

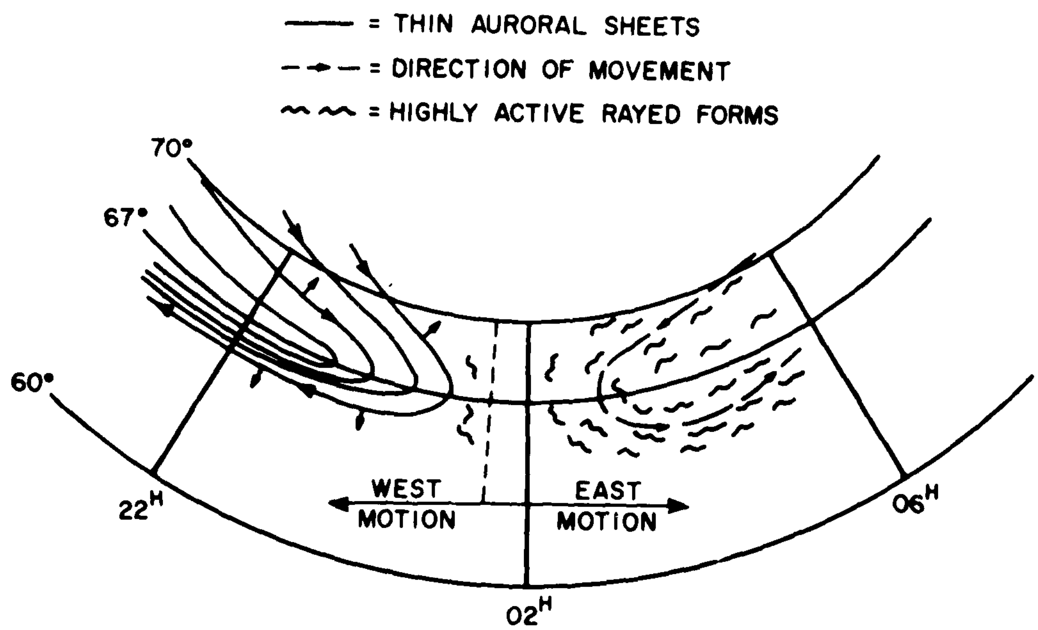


Fig. 5—Auroral morphology of Davis (1960)

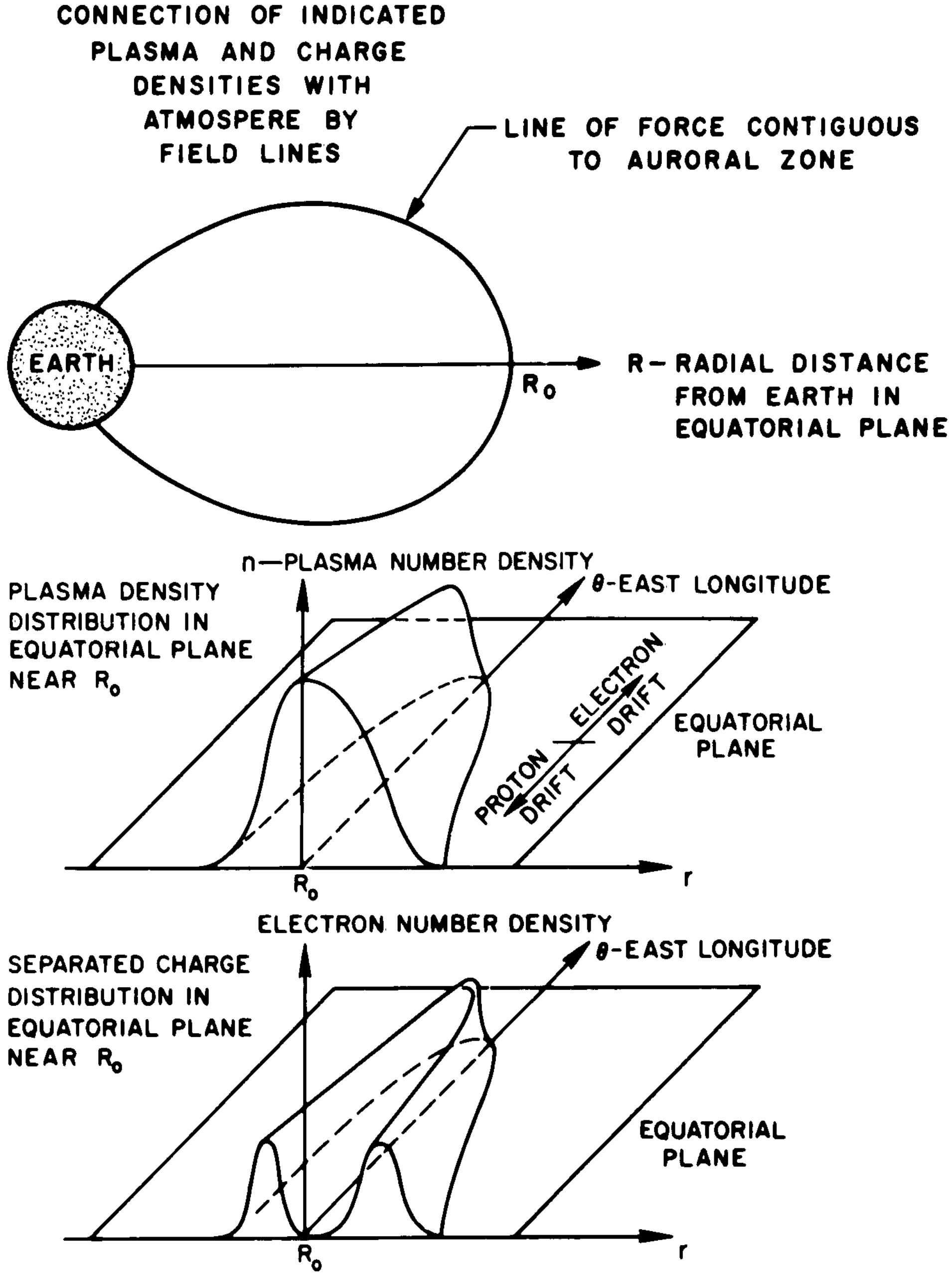


Fig. 6—Charge separation leading to horseshoe auroral arcs

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